On the Mapping between Logical and Physical Topologies

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Question: How to map logical network to physical network such that the demand is satisfied?
Graph Mapping Problem

Demand (Logical network)

Constraints (Physical network)

Question: How to map logical network to physical network such that the demand is satisfied?
Motivations

- Cloud computing
- Social networking
- Smarter networking
- Related work
  - Task-processor assignment
  - Wavelength assignment in optical networks (WDM)
  - Multi-commodity flow optimization
Problem Statement

- **Given**
  - A logical graph $G_L = (V_L, X)$
  - A physical graph $G_P = (V_P, C)$

- **Determine**
  - Node assignment: place 1 logical node at 1 physical node.
  - Flow assignment: find a (multi-path and multi-hop) routing that satisfies the logical requirements and physical capacities.
The two-step approach

- To solve the problem, we take a 2-step approach

Node assignment (NA)

Flow assignment (FA)

NA: Find a feasible node assignment
Feasible NA: under the NA, one can find at least one feasible routing

FA: Find a feasible routing
Feasible routing: satisfies the physical capacities while well meets the logical requirements

When NA is given, FA is the well known Multi-Commodity Flow problem

Our contribution: conditions of NA feasibility and how to find out the feasible NA
Pop quiz

Logical Network

Physical Network

?
No, because the demand cannot be met…

All 1-cut checks are passed
Feasibility testing - the m-cut checks

- **Definitions**
  - **Cut**: a set of edges that partitions a graph into two sides
  - **m-Cut**: a cut that partitions the graph into a set of \( m \) nodes and the other set of \( N-m \) nodes
  - **Cut capacity**: sum of the weight of edges in the cut

- **The m-cut feasibility check**
  - A given NA and a random m-cut
  - If the PHY m-cut capacity \( \geq \) the LOG m-cut capacity, the check is passed
Good news and bad news

- Feasibility condition for NA

  Consider $m$-cut feasibility checks for $m = 1, 2, \ldots, N/2$. If all the checks are satisfied, the particular node assignment is feasible; vice-versa, if a particular node assignment is feasible, it must pass all the $m$-cut checks.

- This condition is shown to be necessary and sufficient using max-flow theory

- There are $C_N^m = \binom{N}{m}$ $m$-cuts!
NA algorithm 1
- Random Assignment Plus Check (RAPC)

- The all-cut RAPC algorithm
  1) Pick up a random node assignment
  2) Check all $m = 1, 2, \ldots, N/2$ cut checks

- Complexity of RAPC
  - If two network can be mapped by some NA
    • The RAPC goes over $k-1$ unfeasible NA’s
    • Reach a feasible NA at $k$-th trial
    • Complexity: $O(2^N)$
  - If two networks cannot be mapped via any NA
    • The RAPC goes over all possible $N!$ NA’s
    • Complexity: $O(N!)$
Catch probability $P_m$

Random generated PHY and LOG of size $N$ and $M$ in the range of 2 – 10

Uniformly distributed edge weights
NA algorithm 2
- Simplified RAPC (1-cut RAPC)

- The 1-cut RAPC algorithm
  1) Pick up a random node assignment
  2) Check the all 1-cut check
  3) If all the 1-cut checks are passed, the NA is asserted to be feasible

- Complexity of 1-cut RAPC
  - If two network can be mapped by some NA
    • Complexity: $O(N)$
  - If two networks cannot be mapped via any NA
    • The RAPC goes over all possible $N!$ NA’s
    • Complexity: $O(N!)$
NA algorithm 3
- Greedy 1-cut mapping (1-cut MA)

- Instead of random assignment, we assign nodes in a greedy manner

- Greedy 1-cut algorithm
  1) Sort the logical nodes in descending order of 1-cut logical capacity \( \beta_1(1) \geq \beta_1(2) \geq \cdots \geq \beta_1(m) \geq \cdots \geq \beta_1(M) \)
  2) Sort the physical nodes in descending order of 1-cut capacity \( \lambda_1(1) \geq \lambda_1(2) \geq \cdots \geq \lambda_1(n) \geq \cdots \geq \lambda_1(N) \)
  3) Starting from \( k = 1 \), map the \( k \)-th PHY node to the \( k \)-th LOG node, if \( \lambda_1(k) \geq \beta_1(k) \); if \( \lambda_1(k) < \beta_1(k) \) stop the algorithm

- If a NA is formed by 1-cut MA, it must be feasible

- Complexity of 1-cut MA = \( O(N) \)
Simulation results - Complexity of the algorithms

- A specific case for illustration
- When M=N=4

<table>
<thead>
<tr>
<th>(N, M) = (4, 4)</th>
<th>Complexity</th>
<th>Error Pr</th>
</tr>
</thead>
<tbody>
<tr>
<td>All-cut RAPC</td>
<td>566</td>
<td>0</td>
</tr>
<tr>
<td>1-cut RAPC</td>
<td>523</td>
<td>5.3%</td>
</tr>
<tr>
<td>1-cut MA</td>
<td>4</td>
<td>5.3%</td>
</tr>
</tbody>
</table>

1-cut MA algorithm is dramatically faster than RAPC algorithms
The high complexity of RAPC is due to the NP-hard nature of the problem
Error-tolerant scenarios → choose 1-cut MA
Error-sensitive scenarios → choose all-cut RAPC
Performance evaluation - Error probability

- How accurate are the algorithms?
- Erroneous decision: an NA is unfeasible, but the algorithm identifies it as feasible
- Error prob. = prob. of making erroneous decisions

All-cut RAPC: $P_e = 0$

1-cut RAPC: $P_e = (1 - \alpha \varepsilon)(1 - P_1)$

1-cut MA: $P_e = (1 - \alpha \varepsilon)(1 - P_1)$

$P_1 = \text{prob} \{\text{the 1-cut check is failed | the NA is unfeasible}\}$

$1 - P_1 = \text{the prob that the 1-cut cannot catch an unfeasible NA}$

How effective is the 1-cut? = How big is $P_1$?
Simulation results - Probability of feasible mapping

- For two randomly generated network, how often can they be mapped (there is at least one NA feasible)?

The more redundancy → the higher prob of feasible mapping

Gain by redundancy is inversely proportional to the size of logical network
Summary

- This paper
  - Presented a novel set of feasibility checks for node assignments based on graph cuts
  - Showed the conditions to be necessary and sufficient.
  - Proposed a simple and fast algorithm for node assignment based on 1-cut

- Possible next steps
  - Dynamic assignment problem
  - Node assignment with capacity consideration
  - Multiple LOG mapping
  - Other interesting issues
Acknowledgment

- This research was supported through participation in the **International Technology Alliance (ITA)** sponsored by the U.S. Army Research Laboratory and the U.K. Ministry of Defense under Agreement Number W911NF-06-3-0001.